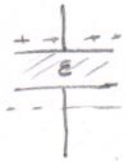
Course 8

Gauss' Law:

$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\varepsilon}$$

for example : capacitor



— $\rho = 0$ but $\mathcal{E} \neq 0$

if $\rho = 0 \not\Rightarrow \mathcal{E} = 0$

there is no need to have $\underline{\rho}$ at the same location of $\mathcal{E}$.

. Uniformly-doped semiconductor in T.E:

In the bulk:

. charge neutrality : $\rho = 0$ $\left\{ \begin{array}{ll} n & N_D^+ \\ p + & N_A^- \end{array} \right.$

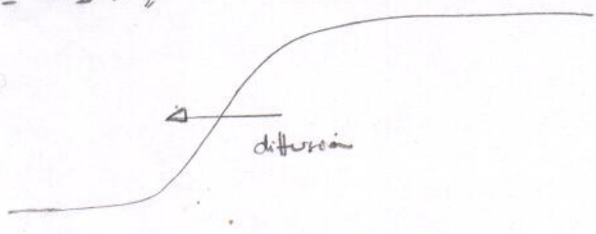
$\therefore$ $\mathcal{E}$ cannot change in space ($d\mathcal{E}/dx = 0$)

$\therefore$ no electric field applied $\Rightarrow$ $\boxed{\mathcal{E} = 0}$

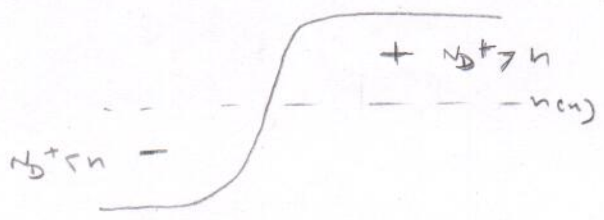
Non-uniformly-doped semiconductor in T.E.

n-type : $\frac{d\mathcal{E}_0}{dx} = \frac{q}{\varepsilon} \cdot (N_D - n_0)$

① if $n(x) = N_D(x)$ $\Rightarrow$ charge neutrality at every single point.



② electrons will flow until $n(x)$ is uniform:



$\Rightarrow$ pull electrons back

$\Rightarrow n(x) \neq N_D(x)$

Using the following expressions:

1) Gauss' Law:

divergence of the electric field is proportional to the volume charge density:

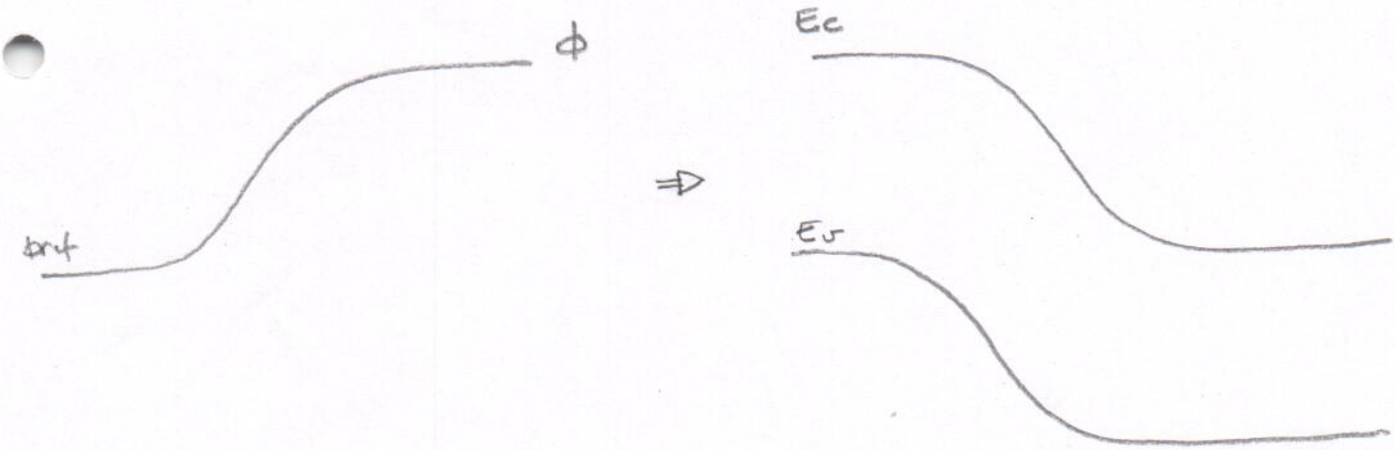
$$\frac{dE}{dx} = \frac{\rho}{\varepsilon}$$

2) $\frac{d\phi}{dx} = -E$ $\Rightarrow$ electric field create a derivative of potential.

3) In an arbitrary reference: $\boxed{E_c + E_{ref} = E_p = -q \cdot \phi}$

thus: $E = -\frac{d\phi}{dx} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_v}{dx}$

the slope of the band structure reflects the shape of the electric potential <u>inverted</u>.



* In TE, $J_e = 0$:

$$J_e = \underbrace{q \cdot \mu_e \cdot n_0\, \mathcal{E}_0}_{\text{drift}} + \underbrace{q \cdot D_e \cdot \frac{dn_0}{dx}}_{\text{diffusion}} = 0$$

in perfect balance.

$$\mathcal{E}_0 = -\frac{D_e}{\mu_e} \cdot \frac{1}{n_0} \cdot \frac{dn_0}{dx}$$

$\Rightarrow$ Einstein relation

$$\begin{cases} \mathcal{E}_0 = -\dfrac{KT}{q} \cdot \dfrac{1}{n_0} \cdot \dfrac{dn_0}{dx} \\[2ex] \mathcal{E}_0 = +\dfrac{KT}{q} \cdot \dfrac{1}{p_0} \cdot \dfrac{dp_0}{dx} \end{cases}$$

$\Rightarrow$ same result can be obtained using $n_0 p_0 = n_i^2$

if we know $n_0, p_0 \Rightarrow \mathcal{E}_0 \Rightarrow \phi_0 \Rightarrow$ In T.E, there is a direct connection between equilibrium carrier concentration and electric field.

$$\mathcal{E}_0 = -\frac{d\phi_0}{dx} = -\frac{KT}{q} \cdot \frac{1}{n_0} \cdot \frac{dn_0}{dx}$$

$$\frac{d\phi_0}{dx} = \frac{KT}{q} \cdot \frac{d\ln(n_0)}{dx}$$

$$\phi_0(x) - \phi(ref) = \frac{KT}{q} \cdot \ln\left(\frac{n_0(x)}{n_0(ref)}\right)$$

* cont.

One possible reference point: $\phi(ref) = 0 \quad n_0(ref) = n_i$

$$\boxed{\phi_0(x) = \frac{KT}{q} \cdot \ln \frac{n_0(x)}{n_i}}$$

$$\begin{cases} n_0 = n_i \cdot \exp\left(\dfrac{q\,\phi_0(x)}{KT}\right) \\[2ex] p_0 = n_i \cdot \exp\left(-\dfrac{q\,\phi_0(x)}{KT}\right) \end{cases}$$

Boltzmann Relations

$\Downarrow$

fundamental law of <u>statistical mechanics</u>

$$n \propto e^{-E_p/KT}$$

$\Rightarrow n \propto e^{-E_p/KT}$ applies also to molecules in the atmosphere, subject to gravitational forces.

$\Rightarrow$ atmosphere gets thinner as we go <u>UP</u>.

$E_p = -q \cdot \phi_0(x)$

Equilibrium carrier concentration

$$\frac{d\mathcal{E}_0}{dx} = \frac{q}{\varepsilon}\left(N_D - n_0\right)$$

$$\frac{d}{dx}\left(-\frac{kT}{q}\cdot\frac{d}{dx}\ln(n_0)\right) = \frac{q}{\varepsilon}\left(N_D - n_0\right)$$

$$\frac{\varepsilon \cdot kT}{q^2}\cdot\frac{d^2}{dx^2}\ln(n_0) = N_D - n_0$$

Solution of this second-order non-linear differential equation gives $\underline{n_0(x)}$, however this is <u>NOT</u> a simple equation.

$$\begin{array}{c} n_0(x) \\ \swarrow \quad \downarrow \quad \searrow \\ \rho(x) \quad \mathcal{E}(x) \quad \phi_0(x) \end{array}$$

Quasi - Fermi levels

#1

In T.E. :

E_F is constant and precisely defines n, p

non - degenerate case:

$$\begin{cases} n_0 = N_C \exp\left(\dfrac{E_F - E_C}{kT}\right) \\ p_0 = N_V \exp\left(\dfrac{E_V - E_F}{kT}\right) \end{cases}$$

a single $\underline{\underline{E_F}}$.

Out of Equilibrium : $n \neq n_0$, $p \neq p_0$ $\therefore$ cannot find a single value of E_F

$\Rightarrow$ Define 2 quasi - fermi levels : $\begin{cases} E_{F_e} \\ E_{F_h} \end{cases}$

for non - degenerate :

$$\begin{cases} n = N_C \cdot \exp\left(\dfrac{E_{Fe} - E_C}{kT}\right) \\ p = N_V \cdot \exp\left(\dfrac{E_V - E_{Fh}}{kT}\right) \end{cases}$$

$\Rightarrow$ Visualization tool : n directly related to the distance of E_{F_e} to E_c

#2

$$n \cdot p = n_i^2 \cdot \exp\left(\frac{E_{Fe} - E_{Fh}}{kT}\right)$$

Let's explore :

$$n = N_c \cdot \exp\left(\frac{E_{Fe} - E_c}{kT}\right)$$

$$\frac{dn}{dx} = \frac{n}{kT} \cdot \left(\frac{dE_{Fe}}{dx} - \frac{dE_c}{dx}\right)$$

$$= \frac{n}{kT} \cdot \left(\frac{dE_{Fe}}{dx} - q \cdot \mathcal{E}\right)$$

$$\mathcal{E} = -\frac{d\phi}{dx}$$

$$\boxed{\mathcal{E} = \frac{1}{q} \cdot \frac{dE_c}{dx}}$$

Calculate current:

$$J_e = q\mu_e \cdot n \cdot \mathcal{E} + qD_e \frac{dn}{dx}$$

$$= q\mu_e \cdot n \cdot \mathcal{E} + q \cdot D_e \cdot \frac{n}{kT} \cdot \frac{dE_{Fe}}{dx} - \frac{q^2 \cdot n \cdot D_e \, \mathcal{E}}{kT}$$

$$\frac{D_e}{kT} = \frac{\mu_e}{q}$$

$$q \cdot n \cdot \mu_e \cdot \mathcal{E}$$

but $\therefore \dfrac{D_e}{\mu_e} = \dfrac{kT}{q}$

$$\begin{cases} J_e = \dfrac{q \cdot D_e}{kT} \cdot n \cdot \dfrac{dE_{Fe}}{dx} = n \cdot \mu_e \cdot \dfrac{dE_{Fe}}{dx} \\ J_h = \mu_h \cdot p \cdot \dfrac{dE_{Fh}}{dx} \end{cases}$$

$\therefore$ gradient of quasi-fermi levels <u>unified</u> driving force for carrier flow as it combines <u>drift</u> and <u>diffusion</u>.

\# 3

$$\cancel{n}\mu_e \frac{dE_{Fe}}{dx} = q \cdot \cancel{n} \cdot v_e$$

$$\frac{dE_{Fe}}{dx} = -\frac{q \cdot v_e}{\mu_e}$$

$\Rightarrow$ if E_{Fe} is flat $\Rightarrow v_e = 0$

$\Rightarrow$ if E_{Fe} has slope $\Rightarrow$ carriers are flowing

$$\frac{dE_{Fh}}{dx} = \frac{q \cdot v_h}{\mu_h}$$

- while recombination is happening, electrons are nearly in equilibrium with <u>lattice</u>
- <u>NOT</u> in equilibrium with holes

$$n \cdot p = n_i^2 \cdot \exp\left(\frac{E_{fe} - E_{fh}}{KT}\right)$$

if E_{fe} is <u>above</u> E_{fh} $\Rightarrow$ $np > n_i^2$ $\Rightarrow$ recombination prevails

else, E_{fe} is below E_{fh} $\Rightarrow$ $np < n_i^2$ $\Rightarrow$ generation prevails.